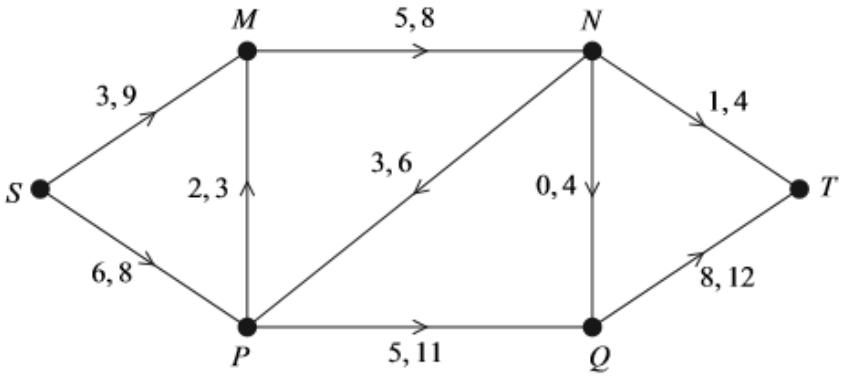
Decision 2 Network Flow Questions

4 [**Figures 3, 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes, with the lower and upper capacities for each pipe in litres per second.



(a) **Figure 3**, on the insert, shows a partially completed diagram for a feasible flow of 10 litres per second from S to T. Indicate, on **Figure 3**, the flows along the edges MN, PQ, NP and NT. *(4 marks)*

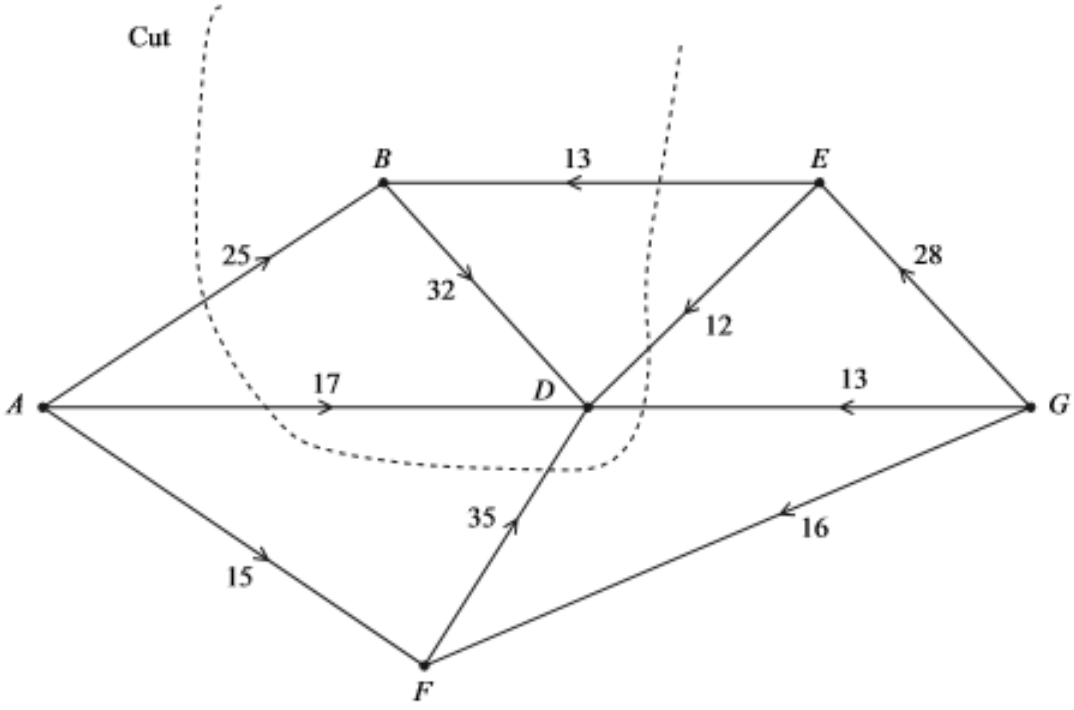
(b) (i) Taking your answer from part (a) as an initial flow, use flow augmentation on **Figure 4** to find the maximum flow from S to T. *(6 marks)*

(ii) State the value of the maximum flow and illustrate this flow on **Figure 5**. *(2 marks)*

(c) Find a cut with capacity equal to that of the maximum flow. *(2 marks)*

4 **[Figures 4 and 5**, printed on the insert, are provided for use in this question.]

The network shows the routes along corridors from the playgrounds A and G to the assembly hall in a school. The number on each edge represents the maximum number of pupils that can travel along the corridor in one minute.



(a) State the vertex that represents the assembly hall. *(1 mark)*

(b) Find the value of the cut shown on the diagram. *(1 mark)*

(c) State the maximum flow along the routes ABD and GED. *(2 marks)*

(d) (i) Taking your answers to part (c) as the initial flow, use a labelling procedure on **Figure 4** to find the maximum flow through the network. *(6 marks)*

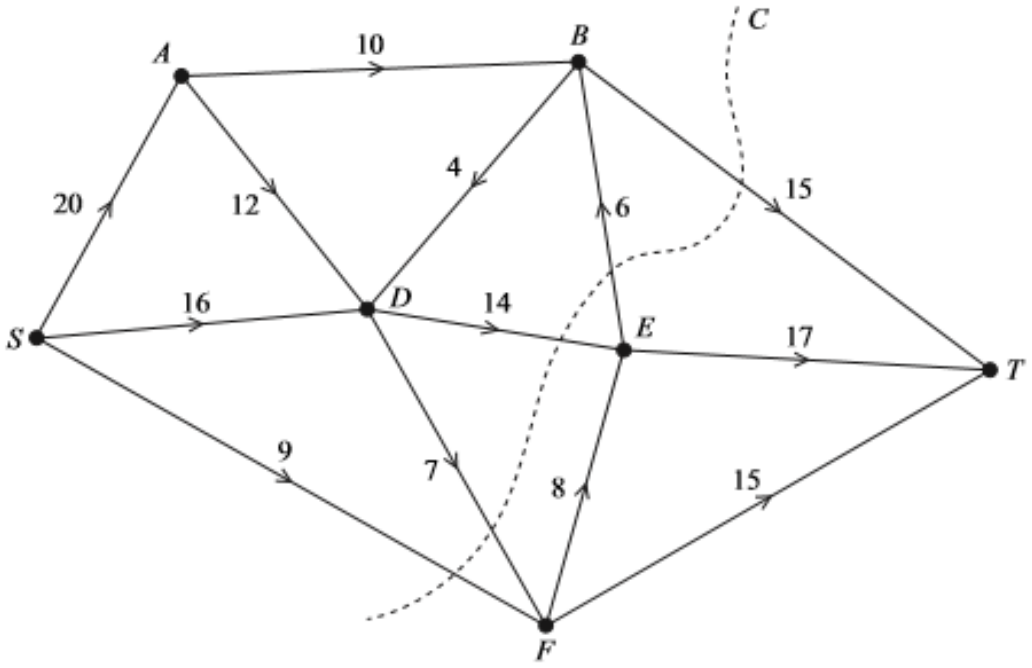
(ii) State the value of the maximum flow and, on **Figure 5**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Verify that your flow is a maximum flow by finding a cut of the same value. *(2 marks)*

(e) On a particular day, there is an obstruction allowing no more than 15 pupils per minute to pass through vertex E. State the maximum number of pupils that can move through the network per minute on this particular day. *(2 marks)*

6 **[Figures 2** and **3**, printed on the insert, are provided for use in this question.]

The diagram shows a network of pipelines through which oil can travel. The oil field is at S, the refinery is at T and the other vertices are intermediate stations. The weights on the edges show the capacities in millions of barrels per hour that can flow through each pipeline.



(a) (i) Find the value of the cut marked C on the diagram. *(1 mark)*

(ii) Hence make a deduction about the maximum flow of oil through the network. *(2 marks)*

(b) State the maximum possible flows along the routes $SABT$, $SDET$ and SFT. *(2 marks)*

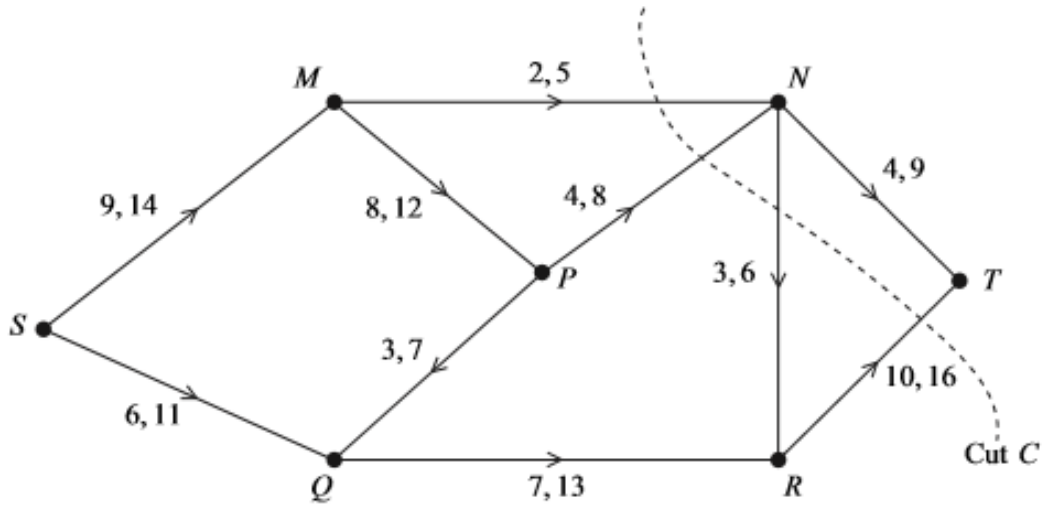
(c) (i) Taking your answer to part (b) as the initial flow, use a labelling procedure on **Figure 2** to find the maximum flow from S to T. Record your routes and flows in the table provided and show the augmented flows on the network diagram. *(6 marks)*

(ii) State the value of the maximum flow, and, on **Figure 3**, illustrate a possible flow along each edge corresponding to this maximum flow. *(2 marks)*

(iii) Prove that your flow in part (c)(ii) is a maximum. *(2 marks)*

6 **[Figures 4, 5 and 6**, printed on the insert, are provided for use in this question.]

The network shows a system of pipes with the lower and upper capacities for each pipe in litres per second.



(a) (i) Find the value of the cut C. *(1 mark)*

(ii) State what can be deduced about the maximum flow from S to T. *(1 mark)*

(b) **Figure 4**, printed on the insert, shows a partially completed diagram for a feasible flow of 20 litres per second from S to T. Indicate, on **Figure 4**, the flows along the edges MP, PN, QR and NR. *(4 marks)*

(c) (i) Taking your answer from part (b) as an initial flow, indicate potential increases and decreases of the flow along each edge on **Figure 5**. *(2 marks)*

(ii) Use flow augmentation on **Figure 5** to find the maximum flow from S to T. You should indicate any flow augmenting paths in the table and modify the potential increases and decreases of the flow on the network. *(5 marks)*

(iii) Illustrate the maximum flow on **Figure 6**. *(2 marks)*

Figure 3 (for use in part (a))

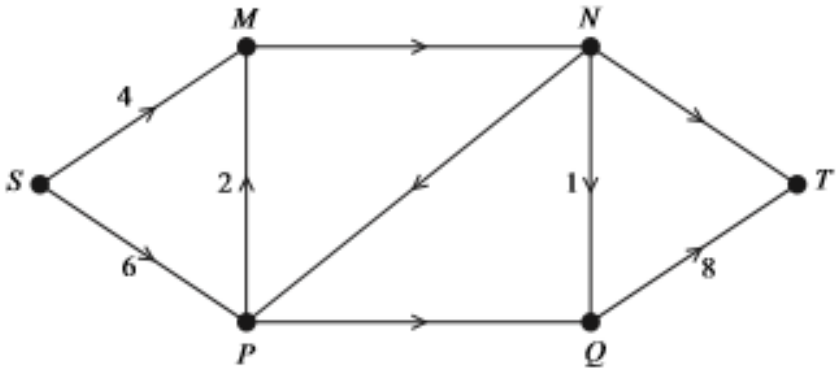
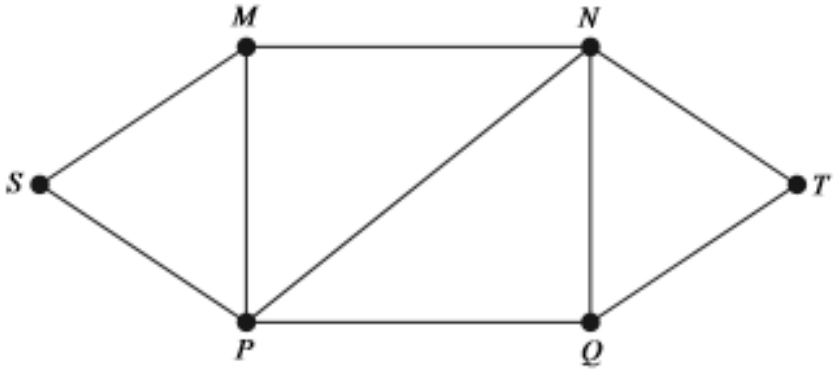


Figure 4 (for use in part (b)(i))



Route	Extra flow

Figure 5 (for use in part (b)(ii))

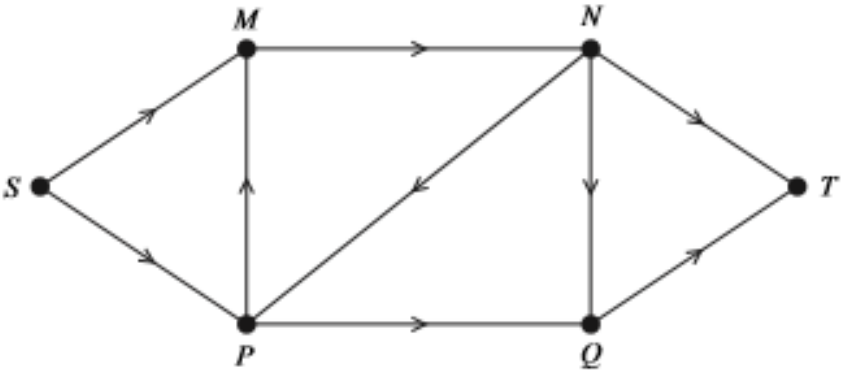
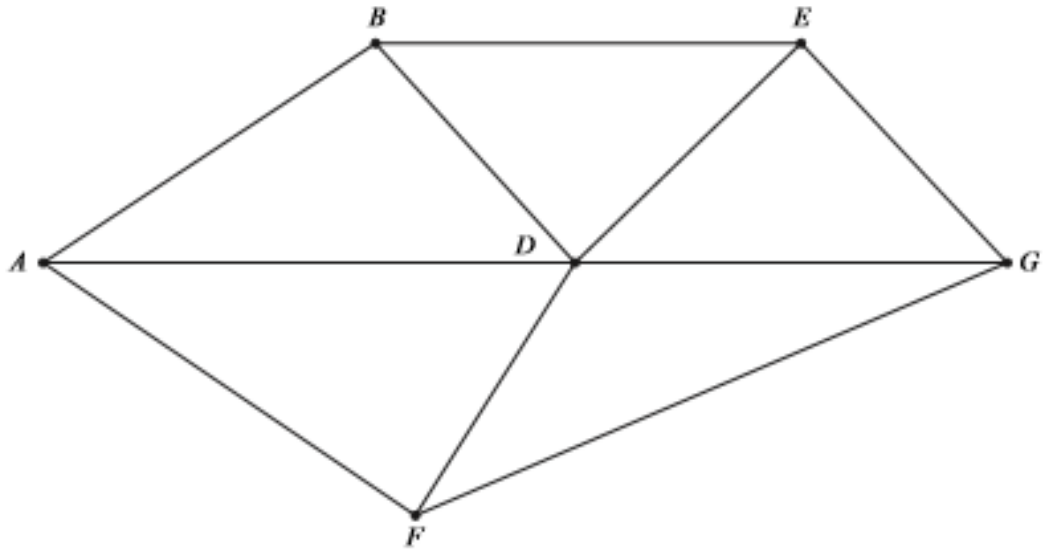


Figure 4 (for use in Question 4 part (d)(i))



Route	Flow
ABD *GED*	

Figure 5 (for use in Question 4 part (d)(ii))

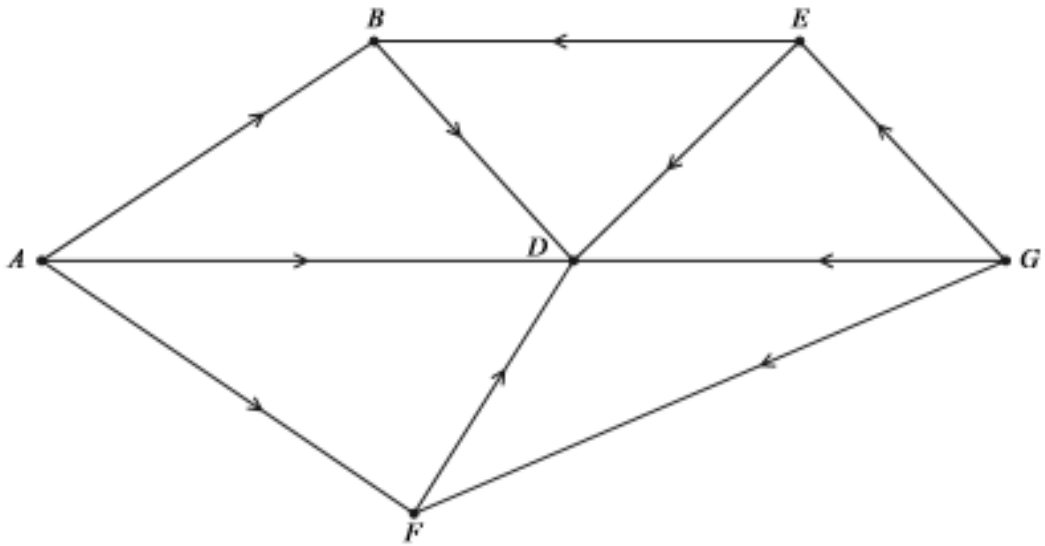
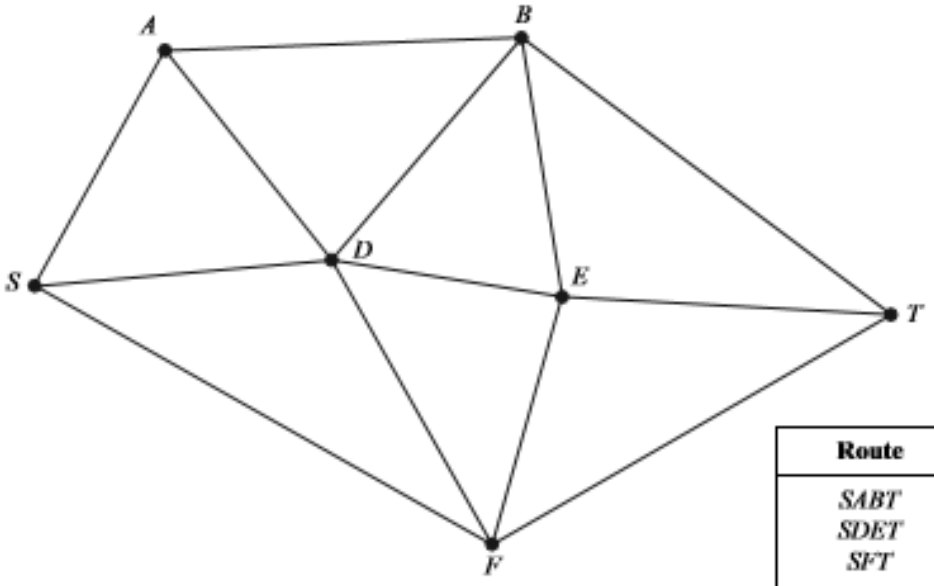


Figure 2 (for use in Question 6(c)(i))



Route	Flow
SABT *SDET* *SFT*	

Figure 3 (for use in Question 6(c)(ii))

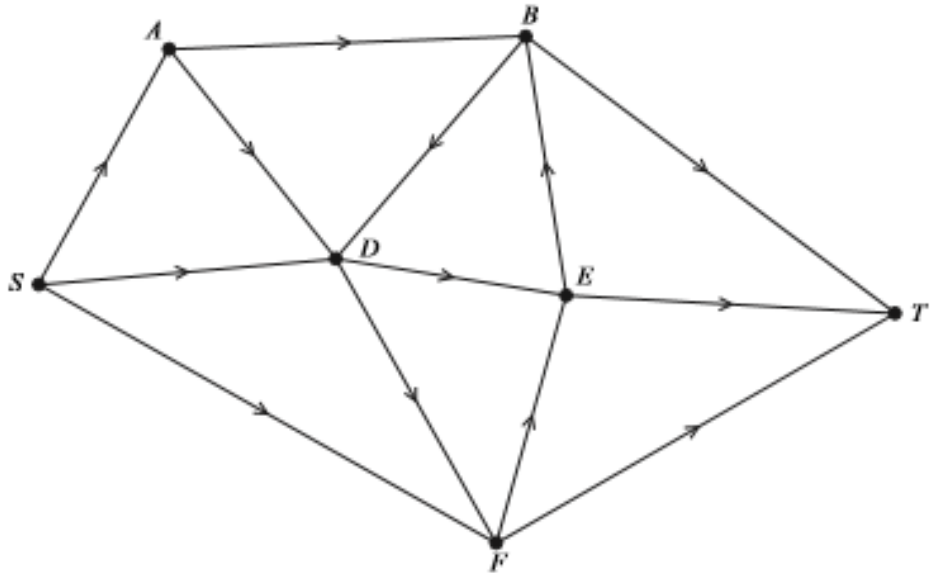


Figure 4 (for use in Question 6)

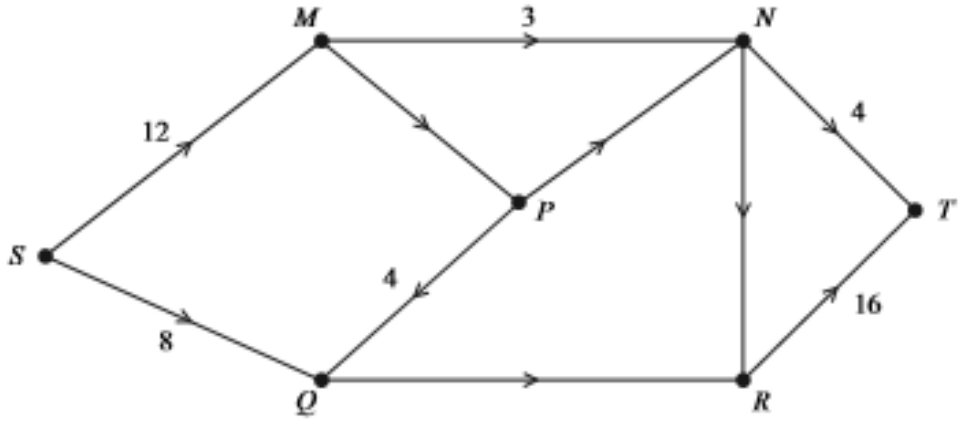
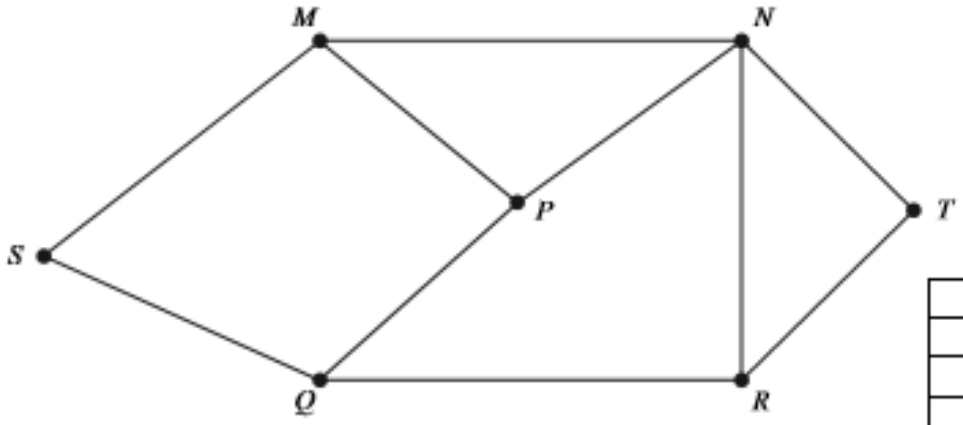


Figure 5 (for use in Question 6)



Path	Flow

Figure 6 (for use in Question 6)